Uncertainty and Entropy in Energy Dependent Economies

Reiner Kümmel(*)

Fluctuating energy prices and the environment's limited capacity of absorbing pollution constitute significant uncertainties for future industrial development. The entropy law, which relates environmental pollution to energy use from earth-internal sources, indicates a method of reducing these uncertainties. The corresponding economic steering mechanisms may involve taxation of energy carriers according to their polluting potential where the tax should fluctuate in such a way that the overall energy price stays constant. Solving a set of differential equations and using the Simplex algorithm in linear programming we look into the question how changing energy inputs and prices have effected industrial growth in the past and how the energy system can be optimized so that pollution becomes minimum in the future.

Introduction

Responsibility of science for the future of life is growing as a result of increasing scientific awareness of the ongoing clash between two strong and hitherto extremely successful traditions: The tradition of nature to create an enormous variety of species on earth endowed with an immense genetic potential ready for adaptation to changing conditions and the tradition of one of these species, man, to push forward his domination over the rest of creation by exploring and using the laws of nature as well as its resources to conquer more and more space on earth for an increasingly comfortable existence in ever growing numbers. In so doing humankind is reducing rapidly the number of living species- about 20% of them will have been exterminated by the year 2000 (1) - and at the same time the composition of air, water, and soil, the inorganic basis for man's existence, is being changed by emissions from his manifold activities. The problems arising from this situation concern basic laws of nature, the dynamics of complex interacting systems, the economic and social behavior of man, and principles of ethics. That is why cooperation of many scientific disciplines is necessary in order to overcome the challenges created by humanity's very success.

Contributions from the natural sciences to this interdisciplinary effort are due in the field of energy. Research on the economic and ecological consequences of energy use is necessary, because energy has been one of the principal factors which have enabled man to spread all over the world and to change it so profoundly. It is the purpose of this paper to indicate how energy has contributed to the generation of material wealth, how this generation has been affected by uncertainties about the availability of energy services, what disturbances are to be expected in the future from energy's ugly twin sister entropy, and

(*) Physikalisches Institut der Universität Würzburg.

what options we many have to maximize the benefits from the use of energy while minimizing the risks that arise from entropy production. From this a proposal results for energy taxation.

Contingency and causality in economic growth

Let us describe the output of an industrial economy by a general production function which depends on capital K (t), labor L (t), energy E (t) and time t:

$$Q(t) = Q\{k(t), L(t), E(t); t] \tag{2.1}$$

The technical definitions of the production factors K, L, E and the output Q and the resulting aggregation principles in terms of work performance and information processing have been given previously (2-4), there it was also shown that it is conceptually acceptable to use their time series from the national accounts and energy balances.

The growth equations of an industrial system are obtained by taking the total time derivative of eq. (2-1):

$$dQ/dt = (\partial Q/\partial k)(dk/dt) + (\partial Q/\partial L)(dL/dt) + (\partial Q/\partial E)(dE/dt) + \partial Q/\partial t. \tag{2.2}$$

We integrate this equation over an infinitesimal time interval dt, divide by Q, multiply on the right hand side by K/K, K/L, E/E, respectively, define the elasticities of production.

$$\alpha'(k,L,E) = (K/Q)(\partial Q/\partial K),\ \beta'\ (K,L,E) = (L/Q)(\partial Q/\partial L),\ \gamma'(K,L,E) = (E/Q)(\partial Q/\partial E) \tag{2.3}$$

and the uncertainty potential

$$P(t) = \partial 1nQ/\partial t \tag{2.4}$$

and arrive at

$$dQ/Q = \alpha' dK/K + \beta' dL/L + \gamma'\ dE/E + P(t)dt \tag{2.5}$$

Eq. (2.5) is the most general equation growth. The term called "uncertainty potential" P(t) is non zero, if there is an explicit time dependence in the production function. It may be seen as a (stochastic) "perturbation" of growth in the sense that it takes into account the influence of human decisions and actions on industrial evolution which go beyond decisions on the magnitude of the inputs k(t), L(t), E(t), think e.g. of discoveries and inventions that lead to innovation breakthroughs.

Pollution U is a principal source of diminishing returns to scale (5). We model this by splitting each of the elasticities of production into two parts: One part describes how growth of a production factor contributes to the growth of output, if the environment's capacity of absorbing pollution is unlimited, the other part indicates the value destroying effect of pollution in a finite environment.

$$\alpha' = \alpha'(K,L,E)p_K(U),\ \beta' = \beta(K,L,E)p_L(U),\ \gamma' = \gamma(K,L,E)\ p_E(U) \tag{2.6}$$

The pollution functions p_F (U), F= K, L, E, are difficult to determine. Plausible analytic forms have been proposed for the case that they may be taken to be the same in all three elasticities of production, i.e. $p_F(u) = p(U)$ [2]; p(U) is a product of pollution functions for the different relevant pollutants, the latter are measured in terms of entropy production, different (natural) purification and critical production rates approximately model the environmental impact of the different pollutants. If one assumes that environmental legislation has advanced so much that it is mandatory to transform all chemical and radioactivate pollution into thermal pollution, the pollution functions have the simple form (6).

$$p(U) = [\exp(U\text{-}Uc)/Uo+1]^{-1}(\exp\text{-}Uc/Uo+1) = P_K = P_L = P_E \tag{2.7a}$$

here thermal pollution is defined as

$$U = R^{-1}\, dS/dt \tag{2,7b}$$

and the entropy production

$$dS/dt = E' / T \tag{2.7c}$$

is given by the global energy input per unit time E', which essentially is transformed into heat at the average temperature T of the environment, R is the space of the biosphere. The purification rate Uo is proportional to the rate at which earth radiates heat into space $Uc = 3 \times 10^{14}$ Watts/TR is the so called heat barrier: If U exceeds Uc, unacceptable climatic changes are likely, present global anthropogeneous heat generation is about 9×10^{12} Watts. p(U) decreases from unity for $U < Uc$ to zero for $U > Uc$.

Combining eqs, (2.5) - (2.7) one obtains the equation of growth with stochastic and deterministic elements: The stochastic elements which are beyond scientific predictability and which perhaps can be handled like statistically fluctuating signals, are the uncertainty potential P (t) and the pollution function p (U). While P (t) takes into account (human) spontaneity in the widest sense, p(U) is an unknown function of capital, labor and energy, because we do not know, how at a given level of pollution $U = Uc$ nature's nonlinear cybernetic circuits, coupled in multiple, complex ways to the economic system, react to a given combination of factor inputs. Thus, analytic solutions of eq (2.5) are possible only, if one can regard P(t) and p(U) as nearly negligible perturbations, i.e. put

$$P(t) \approx 0,\ p(U) \approx 1 \tag{2.8}$$

Then, the output Q is uniquely determined by the inputs of capital K, labor L, and energy E, because of the causal-deterministic laws of technology [2] reigning in industrial production. As a result we have constant returns to scale, i.e.

$$\alpha + \beta + \gamma = 1 \tag{2.9}$$

and the elasticities of production α, ß and y must satisfy the set of partial differential equations (2-4)

$$K\partial\alpha/\partial K + L\partial\alpha/\partial L + E\partial\alpha/\partial E = 0 \tag{2.10a}$$

$$K\partial\beta/\partial K + L\partial\beta/\partial L + E\partial\beta/\partial E = 0 \tag{2.10b}$$

$$L\partial\alpha/\partial L = K\partial\beta/\partial K \tag{2.10c}$$

subject to the restrictions

$$\alpha \geq 0,\ \beta \geq 0\ ,\ 0 \leq \alpha + \beta < 1 \tag{2.11}$$

Eqs. (2.5) - (2.11) are invariant under the transformation to dimensionless variables.

$$k = K/Ko\ ,\ l = L/Lo\ ,\ \ e = E/Eo\ ,\ q = Q/Qo \tag{2.12}$$

where the index 0 refers to a basis year.

Eqs. (2.10) can be solved, if appropriate boundary conditions are known for the elasticities of production. Asymptotically it is reasonable to demand that α goes to zero, if the ratio of energy and labor to capital becomes vanishingly small, and that ß vanishes, if the state of total automation is being approached. Solutions of eqs. (2.10) which satisfy these conditions are

$$\alpha = a_o\, (l + e)/k,\ \beta = a_o\, (c_t\, l/e - l/k); \tag{2.13}$$

$2a_o$ is the capital elasticity in the basis year and $1/ct$ is the energetic efficiency of the capital stock (2-4). Inserting the elasticities of production from eqs. (2.13) and (2.9) into the growth equation (2.5), making the approximations (2.7a) and (2.8) and integrating we obtain the (first) LINEX production function.

$$q_{L1} = q_o \, e \exp [a_o (2 - l/k - e/k) + a_o c_t (l/e - 1)] \tag{2.14}$$

which depends linearly on energy and exponentially on the ratios of labor/capital and energy/capital (3,4)

Price shocks and energy conservation

Let us use the LINEX function to analyze the reaction of the German and American economies to the energy price explosions of the Seventies. (In so doing we assume that the approximations (2.8) are valid for the time period between 1960 and 1981, if this assumption turns out to be unjustified, corrections will have to be made). The three free parameters q_o, a_o and c_t are essentially determined by the ordinary least squares method in multiple linear regression, where the empirical data on factors and output are taken from the national accounts, labor statistics and energy balances of the Unites States of America and the Federal Republic of Germany. They are shown by solid lines in Figs. 1 and 2. More details of the econometric analysis are given in References 4 and 7. The dots and open squares in Figs. 1 and 2 represent the theoretical results obtained with the LINEX function for two different set of parameters. We note that after the first steep recession in the wake of the energy price explosion of the years 1973-1975 the theoretical numbers represented by the dots deviate significantly from reality. However, a change of parameters especially an increase of the energetic efficiency $1/c_t$ in the year 1977 brings theory (open squares) nicely back to an agreement with reality which even survives the second oil crisis in 1979. This change of parameters in 1977 indicates an explicit time dependence of the production function during a time interval which includes the year 1977, and such an explicit time dependence gives rise to a finite uncertainty potential P(t). Assuming in a somewhat crude approximation that at the beginning of the year 1977 the old parameter set was still valid while at the end of 1977 the new set had become effective, we obtain from eq (2.4)

P (t=1977) = 0.016/year for Germany's "Warenproduzierendes Gewerbe"
P (t=1977) = 0.036/year for US "Industries"

In all other years P(t) = 0

We see that it has not been completely justified to neglect the uncertainty potential when integrating the equation of growth. At the same time we realize the appropriate method in how to detect it ex post: When one is forced to change parameters from one set of constants to another one in order to regain agreement between theory and reality, P(t) is in operation. In our case energy conservation measures in response to the energy price explosion were taken and became effective by 1977. The success of the (collective) decision to take these measures and alter the technological characteristics of the production systems made people realize that the dependence on energy was not as rigid as one had assumed before the first oil crisis. This softened the shock of the second oil price hike in 1979.

Energy conservation and pollution abatement

The two past energy crises, despite the economic hardship they carried especially for the developing countries, have been very useful. They taught an important lesson concerning a possible response to the pollution-related uncertainties which represent an increasing threat to economic stability: Substantial energy price increases induce energy conservation (and the associated decrease of pollution), if they are assessable, economic damages by price shocks can be avoided.

Fig. 1 Empirical factor inputs and output and theoretical output in West Germany's industrial sector " Warenproduzierendes Gewerbe" (7).

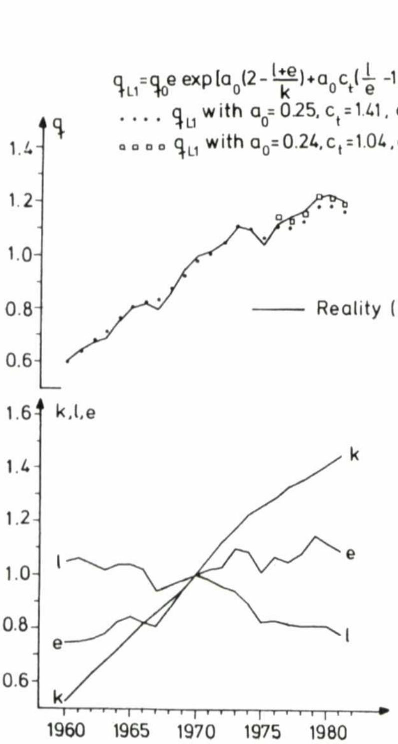
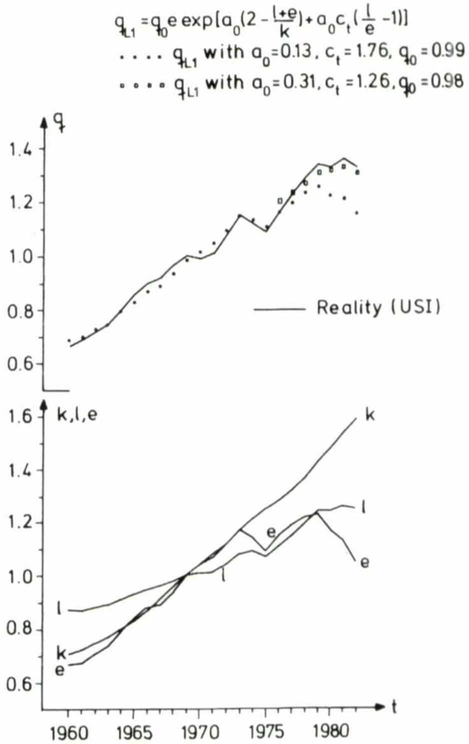


Fig. 2 Empirical factor inputs and output and theoretical output in U.S. "Industries" (7).



The simple model for the interaction between economic activities and environmental pollution, as it is given by the pollution function (2.7) in the equation of growth, may help to illustrate the appropriate strategies for an especially urgent problem of pollution abatement.

The form (2.7a) of the pollution function is always valid, if the impact of one single pollutant dominates that of all others. This way well occur very soon: While the heat barrier looms in a distant future, serious disturbances of world climate have to be expected in the next century because of the carbon dioxide problem. The burning of fossil fuels and the destruction of the forests accumulates carbon dioxide in the atmosphere which lets the sunlight pass through onto earth while blocking partially the radiation of heat from earth into space. This green house effect (which is being reinforced by methan and the common propulsion gases in spray cans) will lead to an increase of the average global temperature. If the production rate of CO2 increases by just 1% per year -the present contribution from China- we have to expect temperature increases between 1.5 and 4.5 centigrades in the moderate altitudes within the next 100 years, whereas at the poles this increase may be twice as large (8). The damages due to coast line flooding because of a possible melting of the West Antarctic Ice Shelf are estimated to be 2.5 x 10^{12} \$)(1971) (9).

If we use the pollution function p(U) of eq. (2.7) in order to model roughly the growth constraining effects of the carbon dioxide problem, we have to identity U with the entropy production density in the subsystem of all carbon atoms, taking it to be proportional to the consumption rate of fossil fuels should be no bad approximation. Uc then may be proportional to the present rate of fossil fuel consumption, although a smaller rate would certainly be safer (8) and U_o would correspond to the rate at which CO2 is being solved in and absorbed by the oceans.

The task is to make pollution U much smaller than its critical value Uc so that p(U) = 1 and growth is not environmentally constrained. This can be achieved in three ways: 1. Provide sufficient, ecologically benign primary energy, a prime candidate for that is solar energy, its price, however, can presently not compete with that of fossil fuels. 2. Decrease the world wide demand for energy services. Since world population is increasing and demands more industrialization, this will be extremely difficult. 3. Industrialize space: the bold vision of the "High Frontier" (10), however, is lacking the massive public financial support it would need for its rapid implementation. 4. Make more efficient use of primary energy. This can be done right away. What ideally may be achieved in some cases can be explored with the help of an optimization procedure which starts from the basic industrial energy demand profiles of The Netherlands, West Germany and Japan and determines the possible amount of energy savings in the industries of these countries allowed by the First and Second Law of Thermodynamics. The results of this optimization are reported here, the details of the method are given in Refs. (11.13).

An energy demand profile shows the heating value of energy, i.e. its enthalpy content, required at a certain temperature or exergy level, where exergy describes the capability of energy to perform mechanical work. The quotient "exergy content divided by enthalpy content" defines the quality Q of an energy carrier (11,12). If the energy carrier is a hot medium of temperature T, its quality can be measured by the Carnot efficiency of an ideal heat engine which operates between the temperature T and de temperature To of the environment:

$$Q = 1 - To / T \tag{4.1}$$

It is convenient to introduce a scaling factor 10 and define by

$$q = 10Q$$

quality levels between 0 (temperature of the environment) and 10 (quality of fossil fuels and electricity). Fig. 3 gives the energy demand profiles of The Netherlands, West Germany and Japan. The left column indicates the enthalpy (in Petajoules) required at different levels of industrial process temperature, and the right column projects this demand on the quality scale where electricity demand at quality 10 has been added. Since the data are already a couple of years old, the actual energy demand profiles may be somewhat different. However, what essentially matters for the saving potential obtained by optimization is the general shape of the profiles, and this should not have changed too much.

The basic idea of the optimization procedure is very simple. It just makes use of the First and Second Law of Thermodynamics which say that energy is never lost, and that its quality is always diminished in processes which are not infinitely slow. Thus, primary energy which has been used to satisfy the demand for a certain energy service characterized by its quality and reused to service part of the total demand at appropriate quality levels. This way primary energy is saved and the associated pollution reduced. Before the first oil price crisis most of secondary energy had been dumped into the environment right away. This should be no longer true as we saw in above. How far one can go until the outmost limits to energy conservation are reached shall be indicated in the following.

The quantity to be minimized is the total amount N(F) of primary energy, called fuel F, required by a given energy demand profile. According to this profile, n(q) enthalpy units have to be delivered to the quality level q from either primary energy F (of quality 10) or from secondary energy S(q') of quality q' by suitable technical devices (furnaces, heat exchangers etc.). The conversion efficiencies associated with these devices are

η (q,F) = fraction of one unit of demand enthalpy (of quality) q generated from one unit of fuel F.

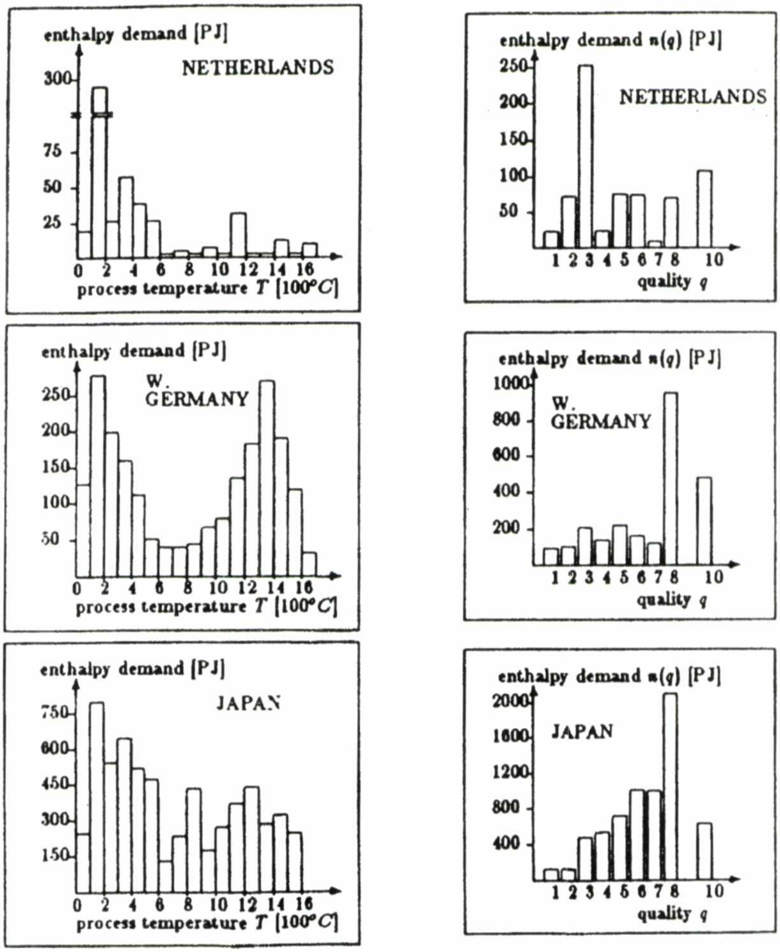
η' (q,q') = fraction of one unit of demand enthalpy (of quality) q generated from one unit of secondary energy of quality q'.

The delivery of energy to a given demand level q results in the generation of secondary energy on other quality levels q' at the secondary energy generation ratios.

ν (q',q,F) = fraction of one unit of secondary energy q' generated during the process of supplying one unit of fuel F to demand level q

ν (q',q, q'') = fraction of one unit of secondary energy q' generated during the process of supplying one unit of secondary energy q'' to demand level q.

Fig. 3 Enthalpy (energy's heating value) demanded at different temperature (T) and quality (q) levels by Dutch, West Germany and Japanese industry (13)



η (q',q)n(q)= amount of secondary energy q' recovered from level q after its demand n(q) has been satisfied by fuel F and/or secondary energy.

These conversion efficiencies and generation ratios are fixed technical data to be taken from the engineering literature. The quantities to be determined by the optimization formalism are the shares x (q,q') or the total amount of secondary energy S(q') on level q' supplied to the demand level q, for all q and q' where $1 \leq q \leq 10$ and $0 \leq q' \leq 9$.

The objective function of minimization, the total demand of fuel F, turns out to be (13).

$$N(F) = \Sigma q\ N(q,F) = \Sigma q\ [n(q) - \Sigma q, \eta'(q,q')\ x\ (q,q')\ S\ (q')]/\eta(q,F) \qquad (4.2)$$

where the total amount S(q') of secondary energy generated on the quality level q' is given by the integral equation

$$S(q') = \Sigma q\ [\mu(q',q)n(q) + \upsilon\ (q',q,F)N(q,F) + \Sigma q\ \upsilon'\ (q',q'q'')\ x\ (q,q'')\ S\ (q')]$$

This integral equation makes the problem highly nonlinear. Only approximate solution, employing iteration schemes, may be possible.

However, for a special case study, which involves heat exchangers, heat pumps and cogeneration, the problem can be made accessible to the methods of linear programming. If we make the following idealizing assumptions which are consistent with our aim to determine the most optimistic possibilities of energy conservation. We assume:

1- All processes occur at the same location in space and time.

2- All secondary energy in exhausts and generated by friction can be neglected, and on a given quality level q all process heat can be recovered as secondary energy of practically the same quality. Thus, we put

$\upsilon = 0 = \upsilon'$, $\mu = \delta qq'$, (where $\delta qq' = 1$, if $q=q'$ and $\delta qq' = 0$ otherwise)
and obtain $S(q') = n(q')$

The actual loss of quality is taken into account by the restriction that the secondary energy $S(q')$ may only be used on lower levels $q<q'$, if one does not provide additional exergy.

Heat exchangers cascade (secondary) heat from higher quality levels down to lower demand levels while heat pumps, which in our model run on electricity, pump heat from lower to higher quality levels, cogeneration power plants produce electricity and process heat up to quality $q = 6$, losing only about 20% of primary energy as waste heat to the environment.

Minimization of the total amount N (F) of primary fuel with the help of the SIMPLEX algorithm (14) determines the 96 free shares $x(q,q')$ of secondary energy $S(q') = n(q')$ which are to be distributed by the three types of conservation devices to the demand levels. 20 inequality restrictions which are essentially a consequence of the First Law of Thermodynamics have to be observed in the optimization procedure. The main result is the following:

If heat exchangers, heat pumps and cogeneration are employed together and if one assumes that the waste heat from electricity can not be recovered as secondary energy, the possible savings of primary fuel turn out to be:

54% in The Netherlands, 36% in West Germany, 59% in Japan.

The considerable differences in the savings potentials are due to the different shapes of the energy demand profiles: West Germany has a very high energy demand at the level q=8 and relatively low demand at lower levels. Therefore lots of direct heating by primary energy is necessary on level 8, and the resulting secondary energy is more than all the lower levels can take, so that quite a fraction is thrown away. The Dutch and Japanese demand profiles allow a much better use of high quality secondary energy on the lower levels.

In our simple model heat exchangers are the most important conservation devices. Sensitivity analysis shows that if one assumes that only 50% of secondary energy can be used, heat pumps and cogeneration become quite important and make up for a great part of the heat exchangers, therefore, the required primary energy just increases by about 10%.

Realization of the savings potentials indicated by optimization would be a significant contribution to environmental protection.

Entropy indices and energy taxes

The third "negative" oil price shock, i.e. the plunge of oil prices from over \$30/barrel to about \$18/barrel, terminated many efforts of substituting oil by renewable (and environmentally benign) energy resources (15) and eliminated the short-term economic incentives of new investments in energy conservation measures. The continuing degradation of the environment, however, makes further substitution and conservation efforts mandatory. In a market economy they can be stimulated quite simply and in complete agreement with the principles of the market: The prices of goods and services have to include the environmental costs associated with their production. This is easily said and hard to do, because it is extremely difficult to determine environmental costs in general. For the energy sector, however, it may be possible to design an appropriate pricing system based on entropy indices for the principal energy carriers. These entropy indices could be constructed in the following way:

For a given carrier of primary energy determine the heat which would be produced, if all chemical and radioactivate pollutants generated by the use of this carrier would be kept out of or eliminated from the biosphere by suitable technical devices. (It is not difficult to identity these devices, at least in idealized form, for the principal pollutants). The entropy production due to this heat production (E') is given by eq. (2.7c) and represents the entropy index basis for assigning energy taxes to the primary energy carriers. The higher the entropy index the higher the tax on the energy carrier. This tax should be refunded in part or totally to the owner or operators of those installations, which partially or totally avoid pollution (save thermal one). The tax may be kept flexible so that in can buffer energy price fluctuations.

There are other and maybe better economic steering mechanisms of environmental protection. In any case, all of them should be explored vigorously so that legislative action can be taken soon. There is not much time left before ecological breakdowns will produce such dramatic economic instabilities that the system of free markets will be in serious jeopardy.

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